$|\psi\rangle$

$\langle\psi|$

$(\cdots\cdots)$

$\langle\psi|\psi\rangle$ → wave packet

Postulate #4

The only result of a precise measurement of the dynamical value of A are of the eigenvalue a_n of the operator $\hat{A}$

$$\hat{A}|\psi\rangle = a_n|\psi_n\rangle$$

A is Hermitian

$$\langle x|\hat{A}\psi\rangle = \langle A\hat{x}|\psi\rangle$$

$$A|\psi_n\rangle = a_n|\psi_n\rangle$$

$$\langle\psi_n|\hat{A}^\dagger|\psi_n\rangle = a_n^* \langle\psi_n|\psi_n\rangle = a_n^*$$

$$\langle\psi_n|\hat{A}|\psi_n\rangle = a_n \langle\psi_n|\psi_n\rangle = a_n$$

$a_n = a_n^*$

Real eigenvalue

corresponding to Hermitian operator

Note $|\hat{A}4\rangle = \hat{A}|4\rangle$

$\langle \hat{A}4| = \langle 4|\hat{A}^\dagger$

Note 1

$\langle x|\hat{A}4\rangle = \underbrace{\langle x|\hat{A}|4\rangle}_{\text{matrix element of } \hat{A}}$

Postulate #5

If a series of measurement is made, the average (expectation value) describes the result.

$\langle \hat{A} \rangle_4 = \frac{\langle 4|\hat{A}|4\rangle}{\langle 4|4\rangle}$

↑

Average

Function of operator

$[f(\hat{A})]^\dagger = f^*(\hat{A}^\dagger)$

The operator finds the equivalent piece in the dual space

- Unit Operator (Identity)

$\hat{I}|\psi\rangle = |\psi\rangle$

- Inverse Operator

$\exists\, \hat{A}, \hat{B} \quad \ni \quad \hat{B}\hat{A} = \hat{A}\hat{B} = \hat{I}$

→ $\hat{B}$ inverse of $\hat{A}$

$\hat{B} = \hat{A}^{-1}$

*** Unitary Operator

A linear operator are said to be unitary

if $\hat{U}^{-1} = \hat{U}^\dagger$

are $\hat{U}\hat{U}^\dagger = \hat{U}^\dagger\hat{U} = \hat{I}$

Projection Operator

$$\hat{\Lambda}^2 = \hat{\Lambda}$$

e.g. $|4\rangle = |\phi\rangle + |\chi\rangle$

$$|\phi\rangle = \hat{\Lambda}|4\rangle$$

$$|\chi\rangle = (\hat{I} - \hat{\Lambda})|4\rangle$$

Postulate # 6

A wave function can be expanded as a linear combination of the eigenfunctions of $\hat{A}$ (corresponding operator of observable A)

$$|4\rangle = \sum_n c_n |4_n\rangle$$

$$\langle 4_m | 4_n \rangle = \delta_{mn}$$

$$c_m = \langle 4_m | 4 \rangle = \sum_n c_n \underbrace{\langle 4_m | 4_n \rangle}_{\delta_{mn}}$$

c_m: overlap with $|4_m\rangle$

Recall one particle system

$$\Psi(\vec{r},t) = \sum_n \left[\int \psi_n^*(\vec{r}')\,\Psi(\vec{r}',t)\,d\vec{r}'\right]\psi_n(\vec{r})$$

$$= \int \left[\sum_n \psi_n^*(\vec{r}')\,\psi_n(\vec{r})\right]\Psi(\vec{r},t)\,d\vec{r}'$$

$$\rightarrow \sum_n \psi_n^*(\vec{r}')\,\psi_n(\vec{r}) = \delta(\vec{r}-\vec{r}')$$

Closure Relation