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FEB 16

$SO(1,3) = SU(2) \oplus SU(2)$

$(0,\frac{1}{2})$ - right hand Weyl

$(\frac{1}{2},0)$ - left hand

$\cancel{\psi_R^\dagger \psi_R}$ not Lorentz invariant

$\psi_R^\dagger \psi_L + \psi_L^\dagger \psi_R$

$$L_{kinetic} = i\psi_R^\dagger \sigma^\mu \partial_\mu \psi_R$$
$$+ i\psi_L^\dagger \bar{\sigma}^\mu \partial_\mu \psi_L$$

$\sigma^\mu = (\mathbb{I}, \sigma^i) \quad \bar{\sigma}^\mu = (\mathbb{I}, -\sigma^i)$

$(\frac{1}{2},\frac{1}{2})$ vector

Dirac Spinor

$\psi = \begin{pmatrix} \psi_L \\ \psi_R \end{pmatrix} \quad \bar{\psi} = (\psi_R^\dagger, \psi_L^\dagger)$

$$\gamma^\mu = \begin{pmatrix} 0 & \sigma^\mu \\ \bar{\sigma}^\mu & 0 \end{pmatrix}$$

$$\gamma^0 = \begin{pmatrix} 0 & \mathbb{I} \\ \mathbb{I} & 0 \end{pmatrix}$$

$$\bar{\psi} = \psi^\dagger \gamma^0$$

$\gamma^\mu \gamma^\nu + \gamma^\nu \gamma^\mu$

$$\{\gamma^\mu, \gamma^\nu\} = 2g^{\mu\nu}$$

$$\gamma^5 = i\gamma^1\gamma^2\gamma^3\gamma^3 = \begin{pmatrix} -\mathbb{I} & 0 \\ 0 & \mathbb{I} \end{pmatrix}$$

$$P_R = \frac{1+\gamma^5}{2} = \begin{pmatrix} 0 & 0 \\ 0 & \mathbb{I} \end{pmatrix}$$

$$P_L = \frac{1-\gamma^5}{2} = \begin{pmatrix} \mathbb{I} & 0 \\ 0 & 0 \end{pmatrix}$$

$$\{\gamma^5, \gamma^\mu\} = 0$$

$$L = i\bar{\psi} \underbrace{\gamma^\mu \partial_\mu}_{\not{\partial}} \psi - m\bar{\psi}\psi$$

EOM

$$(i\not{\partial} - m)\psi = 0$$

$$\{\psi_\alpha(x), \psi_\beta(y)\} = 0$$

Grassmann Algebra

$$G \supset \{\theta_i\}$$

Grassmann variables

$$\theta_i \theta_j = -\theta_j \theta_i$$

$$\theta_i + \theta_j = \theta_j + \theta_i$$

$$a \cdot \theta \in G \quad a \in \mathbb{C}$$

$$\theta + 0 = \theta$$

Most general elem

$$g = a + b\theta$$

$$a, b \in \mathbb{C}$$

$$\theta^2 = \theta^3 = \cdots 0$$

Fermions

$\theta_1 = \psi(x_1)$, $\theta_2 = \psi(x_2)$, ...

Path Integral

$$\int D\psi = \int_{n\to\infty} d\theta_1 \cdots d\theta_n$$

Grassmann Integrals

- Linear $\int d\theta(s\cdot x + t\cdot y)$

$$= s\int d\theta x + t\int d\theta y$$

$s, t \in \mathbb{C}$

$d\theta$ is anti-commuting

$\int d\theta$ map from $\mathbb{G} \to \mathbb{C}$

$\int d\theta \; X \in \mathbb{C}$

$$\int d\theta \, (a+b\theta) = \cancelto{0}{a\int d\theta} + b\overset{\mathbb{C}}{\int d\theta \, \theta}$$

DEF $\int d\theta \, \theta = 1$

$$\int d\theta \, (a+b\theta) = b$$

$$\frac{\partial}{\partial\theta}(a+b\theta) = b$$

$$\int d\theta \, x(\theta) = x'$$

More than one θ_i

$$\int d\theta_1 \cdots d\theta_n \, x = \left(\frac{d}{d\theta_1} \cdots \frac{d}{d\theta_n}\right) x$$

$$\int d\theta_1 \cdots d\theta_n \, \theta_n \cdots \theta_1 = 1$$

Ex.

$A_{12} \in \mathbb{C}$

$$\int d\theta_1 \, d\theta_2 \, e^{-\theta_1 A_{12} \theta_2} = A_{12}$$

$$= \int d\theta_1 \, d\theta_2 \, (1 - \theta_1 A_{12} \theta_2)$$

$$= A_{12} \int d\theta_1 \, d\theta_2 \, \theta_2 \theta_1$$

$$= A_{12} \int d\theta_1 \, \theta_1$$

$$= A_{12}$$

$n\ \theta_i \quad n\ \bar{\theta}_i$

$1 - \bar{\theta}_i A_{ij} \theta_j + \frac{1}{2} \bar{\theta}_i^2 A_{ij} \theta_j^2$

$$\int d\bar{\theta}_1 d\theta_1 \cdots d\bar{\theta}_n d\theta_n \, \mathrm{EXP}(-\bar{\theta}_i A_{ij} \theta_j)$$

$$= \int d\bar{\theta}_1 d\theta_1 \cdots d\bar{\theta}_n d\theta_n \, \mathrm{EXP}(-\bar{\theta}_i A_{ij} \theta_j)$$

$$= \det(A)$$

Path Integral

$\theta_i \rightarrow \psi(x)$

$\bar{\theta}_i \rightarrow \bar{\psi}(x)$

$$\int d\bar{\theta}_1 d\theta_1 \cdots d\bar{\theta}_n d\theta_n \rightarrow \int D\bar{\psi} D\psi$$

Partition Function $Z[\bar{\eta}, \eta]$

$$= \int D\bar{\psi} D\psi \; e^{i \int d^4x [\bar{\psi}(i \gamma - m)\psi + \bar{\eta}\psi + \bar{\psi}\eta + i\epsilon \bar{\psi}\psi]}$$

$$= N \, \mathrm{Exp}\left[i \int d^4x \, d^4y \, \bar{\eta}(y) \times (i\not\partial - m + i\epsilon)^{-1} \eta(x) \right]]$$

$$\int d\bar{\theta}_1 \, d\theta_1 \cdots d\bar{\theta}_n \, d\theta_n \, \mathrm{EXP}\left(-\bar{\theta}_i A \theta_j + \bar{\eta}_i \theta_i + \bar{\theta}_i \eta_i\right)$$

$$= \mathrm{Exp}\left[\bar{\eta}_i \cdot A^{-1}_{ij} \cdot \eta_j\right] \underbrace{\int d\bar{\theta} \, d\theta \, \mathrm{EXP}\left[-(\bar{\theta} - \bar{\eta} A^{-1}) \cdot A \, (\theta - A^{-1}\eta)\right]}_{N}$$

2-PT FUNCTION

$$\langle 0 | T \, \psi(x) \bar{\psi}(y) | 0 \rangle$$

$$= \frac{1}{Z[0,0]} \frac{\partial^2}{\partial \bar{\eta}(x) \partial \eta(y)} \times Z[\bar{\eta}, \eta] \Big|_{\bar{\eta}, \eta = 0}$$

$$= \frac{i}{i\not\partial - m + i\epsilon} \, \delta^4(x-y)$$

Momentum Space

$$\langle 0 | T \psi(x) \bar{\psi}(y) | 0 \rangle$$

$$= \int \frac{d^4p}{(2\pi)^4} \frac{i}{\not{p} - m + i\epsilon} e^{-ip\cdot(x-y)}$$

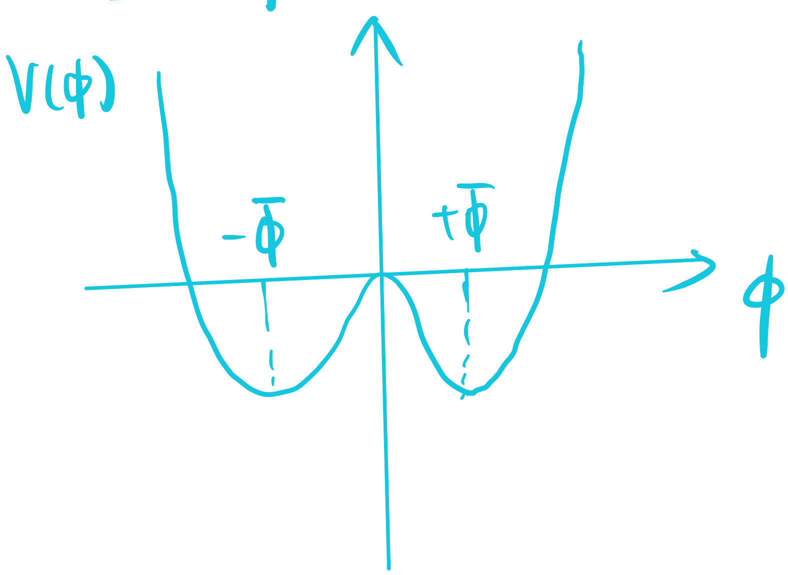
$$= \int \frac{d^4p}{(2\pi)^4} \frac{i(\not{p} + m)}{p^2 - m^2 + i\epsilon} e^{-ip\cdot(x-y)}$$

$$(\not{p} - m)(\not{p} + m) = p^2 - m^2$$

Spontaneous Symmetry Breaking

$SU(3) \times SU(3) \rightarrow SU(3)$

Denegerate Vacuua



Scalar Fields $\phi_n(x)$

$$\phi_n(x) \to \phi_n'(x) = L_{nm}\phi_m(x))$$

$$S(\phi) = S(L\cdot\phi)$$ internal symm

⇒ degenerate sym m

Ex

$\phi \to -\phi$ Z_2

Vacuum Expectation Value (VEV)

$\bar{\Phi}$ is minimum energy config

$\Phi(x) = \bar{\Phi}$ is a constant

Suppose $L\bar{\Phi} \neq \bar{\Phi}$

the vacuum is not unique

$$\int d^4x\, V = -S(\bar{\Phi}) = -S(L\bar{\Phi})$$

Symmetry vacuum of same energy

but nature chose one true vacuum

and the symmetry is braking

Linear combination of va cuum

Spon symm breaking

in infinite only

$-\bar{\phi}$ $|vac, -\rangle$

$+\bar{\phi}$ $|vac, +\rangle$

$\langle vac, + | \hat{H} | vac, + \rangle = \langle vac, - | \hat{H} | vac, - \rangle$

$\equiv a$ (real)

$\langle vac, + | \hat{H} | vac, - \rangle = \langle vac, - | \hat{H} | vac, + \rangle$

$\equiv b$ (real)

$$H = \begin{pmatrix} a & b \\ b & a \end{pmatrix}$$

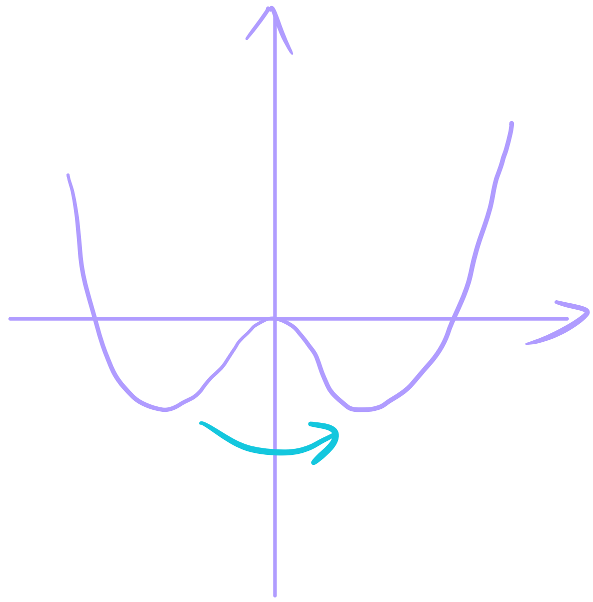
$$|vac,+\rangle \pm |vac,-\rangle \quad \text{Energies } a \pm |b|$$

the actually vacuum preserves symmetry

$$\mathbb{Z}_2 : \phi \to -\phi$$

$$|vac,+\rangle \to |vac,-\rangle$$

the actual vacuum is the lin combo of vacuua



$|b| \propto e^{-c \cdot V}$ ← const ← volum

$V \to \infty \quad |b| \to 0$

Quantum tunneling

Spon symmetry breaking

only infinite space time

If in finite spacetime.

e.g. sphere

quantum tunneling

Some very small perturbation

preserves symmetry

no spont symm breaking

horizon volume.

infinite?

Se(3) order 196