Isospin Symmetry

233A LEC2

$m_p \approx m_n$

$m_p \sim 1\,\text{GeV}$

$\Delta(m_n, m_p) \sim \text{meV}$

QCD (strong force) treats u & d same (symmetry)

EM - - - - different

- proton $\sim uud$
  neutron $\sim udd$

$$|N\rangle = \begin{pmatrix} |p\rangle \\ |n\rangle \end{pmatrix}$$

$$|N\rangle \to U|N\rangle$$

$$\begin{pmatrix} |p\rangle \\ |n\rangle \end{pmatrix} \to \begin{pmatrix} U_{11} & U_{12} \\ U_{21} & U_{22} \end{pmatrix} \begin{pmatrix} |p\rangle \\ |n\rangle \end{pmatrix}$$

$$U^\dagger U = \mathbb{I}$$

$$U \in SU(1)$$

$$U^\dagger U = \mathbb{I}$$
$$\det U = 1$$

Aside

$$\begin{pmatrix} |e,\uparrow\rangle \\ |e,\downarrow\rangle \end{pmatrix} \to U_{\text{space-time}} \times \begin{pmatrix} |e,\uparrow\rangle \\ |e,\downarrow\rangle \end{pmatrix}$$

Invariance

$U^{-1} H U = H$ ↙ Hamiltonian

Show that $m_n = m_p$

$m_p \equiv \langle p | H | p \rangle$

$= \underbrace{\langle p | U^{\dagger} \underbrace{U H U^{\dagger}}_{H} U | p \rangle} \rightarrow m_p = \langle n | H | n \rangle = m_n$

$U = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$ $\det U = +1$

$U|p\rangle = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} = |n\rangle$

SU(2)

$U_{isospin} = \mathbb{I} + i \sum_{a=1}^{3} \delta\phi_a T^a$ ↑ generators of SU(2)

$T^a = \frac{1}{2} \tau^a$ ↑ Pauli Matrices

(i) $U^\dagger U = \mathbb{I}$

$$\Rightarrow i \sum_{\alpha=1}^{3} \delta\varphi_\alpha [T^{\alpha\dagger} - T^\alpha] = 0$$

$$\Rightarrow T^{\alpha\dagger} = T^\alpha \quad \Rightarrow \text{Hermetian}$$

(ii) $\det U_{isospin} = 1$

$$U_{isospin} = e^{i\delta\phi_\alpha T^\alpha}$$

$$\det e^A = e^{\mathrm{Tr}A}$$

$$\det U_{isopin} = e^{i\delta\phi_\alpha \mathrm{Tr}[T^\alpha]} = 1$$

$$\mathrm{Tr}\,T^a = 0 \rightarrow \text{Traceless}$$

$\Rightarrow$ Three independent, traceless hermitian.

$$\tau_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad \tau_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \quad \tau_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

States Eigenvalue

$|\vec{T}|^2 = T^a T^a \ldots\ldots I(I+1)$

T_3 I_3

	I	I_3
$\|p\rangle$	$\frac{1}{2}$	$\frac{1}{2}$
$\|n\rangle$	$\frac{1}{2}$	$-\frac{1}{2}$

$$T_3|n\rangle = \frac{1}{2}\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}\begin{pmatrix}0\\1\end{pmatrix} = -\frac{1}{2}\begin{pmatrix}0\\1\end{pmatrix} = -\frac{1}{2}|n\rangle$$

Ex:

	I	I_3
pp	1	1
$\frac{1}{\sqrt{2}}(pn+np)$	1	0
nn	1	-1
$\frac{1}{\sqrt{2}}(pn-np)$	0	0

$|\text{spin } \tfrac{1}{2}\rangle \otimes |\text{spin } \tfrac{1}{2}\rangle$

$= |\text{spin } 1\rangle \oplus |\text{spin } 0\rangle$

Pions for Isotriplets

$$T_i |\pi_j\rangle = i \varepsilon_{ijk} |\pi_k\rangle$$

$i = 1, 2, 3$

Define:

$$|\pi^\pm\rangle = \frac{1}{\sqrt{2}} \left(|\pi_1\rangle \pm i |\pi_2\rangle \right)$$

$$T_3 |\pi^\pm\rangle = \frac{1}{\sqrt{2}} \left[\varepsilon_{312} |\pi_2\rangle \pm (-1) \varepsilon_{321} |\pi_1\rangle \right]$$

$\varepsilon_{312} = 1$

$\varepsilon_{321} = -1$

$$= \frac{1}{\sqrt{2}} \left[|\pi_2\rangle \pm |\pi_1\rangle \right]$$

$$= \pm |\pi^\pm\rangle$$

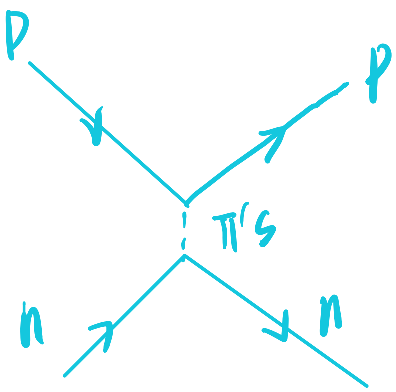
Eigenvalue ± 1

under the action of generator T_3

$$T_3 |\pi_3\rangle = i\epsilon_{33k} |\pi_k\rangle$$

$$= 0$$

	I	I_3
π^+	1	+1
π^0	1	0
π^-	1	−1



Effective Lagrangian

$$\mathcal{L} \propto \bar{N}_i (T^a)_{ik} \pi^a N_k$$

$i = 1, 2$ $\quad N_1 = p$

$a = 1, 2, 3$ $\quad N_2 = n$

$2 \otimes 2 = 3 \oplus 1$

Pion can transform like a fundamental

$N \to UN \qquad \bar{N} \to \bar{N} U^\dagger$

$\vec{T} \cdot \vec{\pi} \to U \vec{T} U^\dagger \vec{\pi}$

$SU(2)$ Antifundamental

= fundamental

$\mathcal{L} \to \bar{N} \cancel{U^\dagger U} \vec{T} \cancel{U^\dagger} \vec{\pi} \cancel{U} N$

$= \bar{N} \vec{T} N = \mathcal{L}$

Lagrangian goes to itself under $SU(2)$

$$T^a \pi^a = \begin{pmatrix} \pi^3 & \sqrt{2}\, \pi^- \\ \sqrt{2}\, \pi^+ & -\pi^3 \end{pmatrix}$$

$$\mathcal{L} \sim (\bar{p}\ \bar{n}) \begin{pmatrix} \pi^3 & \sqrt{2}\, \pi^- \\ \sqrt{2}\, \pi^+ & -\pi^3 \end{pmatrix} \begin{pmatrix} p \\ n \end{pmatrix}$$

$$\sim \sqrt{2}\, \bar{p} n \pi^+ + \sqrt{2}\, \bar{n} p \pi^- + (\bar{p} p - \bar{n} n) \pi^0$$

$\pi^+ \to e^{ie\alpha}\pi^+$ $\quad \pi^- \to e^{-ie\alpha}\pi^-$

$\bar{p} \to \bar{p}\, e^{-ie\alpha}$

Strangeness

"V-particles"

- K-mesons, Λ-hyperons

① $\pi^- + p \to K^0 + \Lambda$ $\quad$ Big cross section

$\sigma(\pi^- p \to K^0 \Lambda) \approx 1\,mb = 10^{-27}\,cm^3$

$\sigma(\pi^- p \to \text{anything}) \approx 40\,mb$

$E_\pi \approx 1.5\,GeV$

$\sigma = (fm)^2 \sim (10^{-13}\,cm)^2 = 10^{-26}\,cm^2$

$1\,fm \sim \frac{1}{200\,meV} \sim \frac{1}{m_\pi}$

$\hbar c = 1 = 200\,MeV\,fm$

(i) $\pi^- + p \rightarrow K^0 + \Lambda$ follow conservation of strangeness

observed

(ii) $\pi^- + p \rightarrow K^- + p$ violate strangeness

never observed

Puzzle 2

$\Lambda \rightarrow p + \pi^-, \; n + \pi^0$

way too slow

SU(3)