$$\rho = \frac{M}{\frac{4}{3}\pi R^3} = \text{const}$$

$$\vec{g}(\vec{r}) = g(r)\,\vec{e_r}$$

$$g(r) = \begin{cases} -G\frac{M}{r^2} & r \geq R \\ -\frac{GMr}{R^3} & r < R \end{cases}$$

$$\vec{g} = -\nabla\phi \qquad \phi_1 - \phi_2 = -\int_{\vec{r_1}}^{\vec{r_2}} \vec{g}\cdot d\vec{s}$$

$$g(r) = -\frac{d\phi}{dr}$$

$$\phi(r) = \begin{cases} -\frac{GM}{r} & r \geq R \\ \frac{GM}{2R^3}r^2 - \frac{3}{2}\frac{GM}{R} & r \leq R \end{cases}$$

Self Energy $U = \frac{1}{2}\int -\frac{G\rho(\vec{r})\rho(\vec{r}\,')\,d^3\vec{r}\,d^3\vec{r}\,'}{|\vec{r}-\vec{r}\,'|}$

$$U = \frac{1}{2}\int \rho(\vec{r})\,d^3\vec{r} \times \underbrace{\int -\frac{G\rho(\vec{r}\,')\,d^3\vec{r}\,'}{|\vec{r}-\vec{r}\,'|}}_{\phi(\vec{r})}$$

$$= \frac{1}{2}\int \rho(\vec{r})\,\phi(\vec{r})\,d^3\vec{r}$$

$$= \frac{1}{2}\int_0^R \rho\,\phi(\vec{r})\,4\pi r^2 dr$$

$$= \frac{1}{2}\rho\,4\pi\int_0^R \phi(\vec{r})\,r^2 dr$$

$$= \frac{1}{2}\rho\,4\pi\int_0^R \left[\frac{GM}{2R^3}r^2 - \frac{3}{2}\frac{GM}{R}\right] r^2 dr$$

$$= \frac{1}{2}\rho 4\pi\left[\int_0^R \frac{GM}{2R^3} r^4 dr - \int_0^R \frac{3}{2}\frac{GM}{R} r^2 dr\right]$$

$$= \frac{1}{2}\rho 4\pi\left[\frac{GM}{2R^3}\frac{r^5}{5}\Big|_0^R - \frac{\cancel{3}}{2}\frac{GM}{R}\frac{r^3}{\cancel{3}}\Big|_0^R\right]$$

$$= \frac{1}{2}\rho 4\pi\,GM\left(\frac{R^5}{10R^3} - \frac{R^3}{2R}\right)$$

$$= \frac{1}{2}\rho 4\pi\,GM\left(\frac{R^2}{10} - \frac{R^2}{2}\right)$$

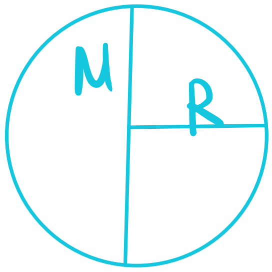
$$= \frac{1}{2}\rho 4\pi\,GM\,\frac{R^2 - 5R^2}{10}$$

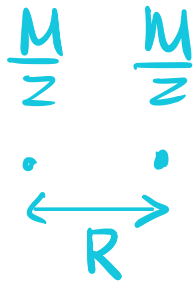
$$= \frac{1}{2}\rho 4\pi\,GM\left(-\frac{2}{5}\right)R^2$$

$$= -\frac{1}{5}4\pi\rho GMR^2$$

$$= -\frac{1}{5}\cancel{4\pi}\frac{M}{\cancel{\frac{4}{3}\pi R^3}\,R}GM\cancel{R^2}$$

$$= -\frac{3}{5}\frac{GM^2}{R}$$





$$U = -\frac{G\left(\frac{M}{2}\right)^2}{R}$$

$$\sim -\frac{GM^2}{R}$$

Pressure $P \sim \frac{F}{R^2} \sim \frac{GM^2}{R^4}$

$$P = nk_BT = \frac{GM^2}{R^4}$$

$$E \cong Mc^2 - \frac{GM^2}{R}$$

$\rho_c \Rightarrow E=0$

$\rho>\rho_c \quad E<0$

$\rho<\rho_c \quad E>0$